

AL-TP-1992-0046

AD-A258 622



ARMSTRONG
LABORATORY

**PROBABILISTIC STUDENT MODELING
WITH KNOWLEDGE SPACE THEORY**

Michael Villano
Charles Bloom

DTIC
ELECTE
DEC 18 1992
S B D

Honeywell Sensor and System Development Center
3660 Technology Drive
Minneapolis, MN 55418

HUMAN RESOURCES DIRECTORATE
TECHNICAL TRAINING RESEARCH DIVISION
Brooks Air Force Base, TX 78235-5000

October 1992

Interim Technical Paper for Period February 1991 - February 1992

Approved for public release; distribution is unlimited.

92-31898



16p8

92 11 17 088

AIR FORCE MATERIEL COMMAND
BROOKS AIR FORCE BASE, TEXAS 78235-5000

NOTICES

This technical paper is published as received and has not been edited by the technical editing staff of the Armstrong Laboratory.

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely Government-related procurement, the United States Government incurs no responsibility or any obligation whatsoever. The fact that the Government may have formulated or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication, or otherwise in any manner construed, as licensing the holder, or any other person or corporation; or as conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

The Office of Public Affairs has reviewed this paper, and it is releasable to the National Technical Information Service, where it will be available to the general public, including foreign nationals.

This paper has been reviewed and is approved for publication.


JIMMY L. FLEMING
Project Scientist


JAMES W. PARLETT, Major, USAF
Chief, Intelligent Training Branch


RODGER D. BALLENTINE, Colonel, USAF
Chief, Technical Training Research Division

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE October 1992	3. REPORT TYPE AND DATES COVERED Interim - February 1991 - February 1992		
4. TITLE AND SUBTITLE Probabilistic Student Modeling with Knowledge Space Theory		5. FUNDING NUMBERS C - F33615-91-C-0002 PE - 62205F PR - 1121 TA - 09 WU - 81		
6. AUTHOR(S) Michael Villano Charles Bloom				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Honeywell Sensor and System Development Center 3660 Technology Drive Minneapolis, MN 55418		8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Armstrong Laboratory Human Resources Directorate Technical Training Research Division Brooks Air Force Base, TX 78235-5000		10. SPONSORING/MONITORING AGENCY REPORT NUMBER AL-TP-1992-0046		
11. SUPPLEMENTARY NOTES Armstrong Laboratory Technical Monitor: Jimmy L. Fleming, (512) 536-2034				
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.		12b. DISTRIBUTION CODE		
13. ABSTRACT (Maximum 200 words) This article presents Knowledge Space Theory (Falmagne and Doignon) as the foundation for a probabilistic student model to be imbedded in an Intelligent Tutoring System (ITS). Applications to typical ITS student modeling issues such as knowledge representation, adaptive assessment, curriculum representation, advancement criteria, and student feedback are discussed. Several factors contribute to uncertainty in student modeling such as careless errors and lucky guesses, learning and forgetting, and unanticipated student response patterns. However, a probabilistic student model can represent uncertainty regarding the estimate of the student's knowledge and can be tested using empirical student data and established statistical techniques.				
14. SUBJECT TERMS Artificial neural networks Intelligent computer aided instruction Intelligent tutoring systems Student modeling			15. NUMBER OF PAGES 20	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL	

CONTENTS

Introduction	1
Knowledge Representation in the Student Model	2
Student Model Construction	3
Applications of the Student Model	5
Evaluation of a Probabilistic Student Model	9
Discussion	9
References	10

DTIC

Accession For		%
NTIS GRA&I	<input checked="" type="checkbox"/>	
DTIC TAB	<input type="checkbox"/>	
Unannounced	<input type="checkbox"/>	
Justification		
By		
Distribution/		
Availability Codes		
Dist	Avail and/or Special	
A-1		

PREFACE

The mission of the Intelligent Training Branch of the Technical Training Research Division of the Human Resources Directorate of the Armstrong Laboratory (AL/HRTI) is to design, develop, and evaluate the application of artificial intelligence (AI) technologies to computer-assisted training systems. The current effort was undertaken as part of HRTI's research on intelligent tutoring systems (ITS) and ITS development tools. The work was accomplished under work unit 1121-09-81, Application of Artificial Neural Networks to Modeling Student Performance. The proposal for this research was solicited using a Broad Agency Announcement.

Probabilistic Student Modeling with Knowledge Space Theory¹

Michael Villano and Charles Bloom

Honeywell Sensor and System Development Center
3660 Technology Drive
Minneapolis, MN 55418

Abstract

This article presents Knowledge Space Theory (Falmagne and Doignon) as the foundation for a *probabilistic* student model to be imbedded in an Intelligent Tutoring System (ITS). Applications to typical ITS student modeling issues such as knowledge representation, adaptive assessment, curriculum representation, advancement criteria, and student feedback are discussed. Several factors contribute to uncertainty in student modeling such as careless errors and lucky guesses, learning and forgetting, and unanticipated student response patterns. However, a probabilistic student model can represent uncertainty regarding the estimate of the student's knowledge and can be tested using empirical student data and established statistical techniques.

Introduction

The student model in an Intelligent Tutoring System provides support for the following functions: adaptively assessing the student's mastery of the course material, representing the student's progress through the curriculum, selecting the appropriate level of hinting and explanation, determining advancement and facilitating student feedback. Ideally, the student model should maintain as much information about the student's knowledge as is necessary to meet the demands of the ITS. In addition to dynamically adapting to new information obtained from the student's responses during an individual's interaction with the ITS, the student model should also be capable of utilizing assessment experience obtained from a population of students. The motivation for a *probabilistic* student model stems from the need to represent uncertainty regarding the estimate of the student's knowledge. Several factors contribute to uncertainty in student modeling such as careless errors and lucky guesses in the student's responses, changes in the student knowledge due to learning and forgetting, and patterns of student responses simply unanticipated by the designer of the student model.

The purpose of this paper is to consider the application of Knowledge Space Theory (Falmagne and Doignon) as a probabilistic student model imbedded in an Intelligent Tutoring System (ITS). An ITS for high school mathematics is in the planning stages using KST (Falmagne, personal communication, November 1991).

¹This research was supported in part by Armstrong Laboratory, Human Systems Division (AFSC), United States Air Force, Brooks AFB, TX 78235-5600 under contract number F33615-91-C-0002.

Knowledge Space Theory was developed primarily for adaptive, computerized knowledge assessment. Therefore, some conjecture will be necessary to infer how the various functions of a student model could be handled by this theory.

Knowledge Representation in the Student Model

Knowledge Space Theory

A comprehensive theory of knowledge representation and assessment has been developed by Falmagne, Doignon and their associates (Falmagne and Doignon, 1985; Falmagne, Koppen, Villano, Doignon and Johannesen, 1990). In their Knowledge Space Theory (KST), the basic unit of knowledge is an *item*. Each item can be in the form of a problem or an equivalence class of problems that the student has to solve. An item may also be presented as a task which the student has to perform if the goal is to assess procedural knowledge. Thus, a body of knowledge is characterized by a set of items called the *domain*. The following items will be used as examples throughout the text:

- a. $4 \times 7 = ?$ b. $1/4 \times 1/7 = ?$ c. $0.4 \times 7 = ?$ d. 40% of 7 = ?

The student's *knowledge state* is defined as the collection of items the student is capable of solving. For example, the knowledge state $\{a, b, d\}$ corresponds to a student who can solve Items a, b and d but who could not solve Item c . Not all subsets of items are considered to be feasible states. For example, if a student is capable of solving the percentage problem (Item d) then we may be able to infer that the student can perform single-digit multiplication (Item a) and thus, any state that contained Item d would contain Item a . We also might not expect to find a student who could answer Item d and none of the other items, thus $\{d\}$ would not be considered a feasible state. The collection of all feasible states is called the *knowledge structure*. A knowledge structure must contain the null state $\{\}$ which corresponds to the student who fails all the items, and the domain which corresponds to the student who has mastered all the items. An example knowledge structure for the four items a, b, c, d appears in Figure 1.

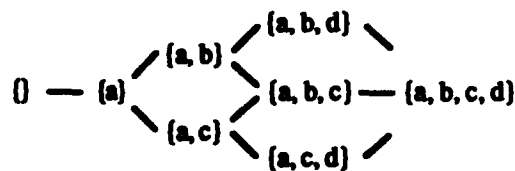


Figure 1. Example knowledge structure.

An important special case of a knowledge structure occurs when the collection of states is closed under union. That is: if two subsets of items are states in the knowledge structure then their union is also a state. A knowledge structure satisfying this condition is called a *knowledge space*. In Figure 1, notice that $\{a, b, d\} \cup \{a, c, d\} = \{a, b, c, d\}$, which is a state in the knowledge space. An additional and stronger condition on a knowledge space involves the assumption that any knowledge state is on a "learning path," consisting in an increasing sequence of

Probabilistic Student Model

states. Beginning with the null state and finishing with the full set of items, any state in the path different from the null state contains exactly one more problem than the preceding state. Such a learning path is called a *gradation*. The following chain of states illustrates one of the gradations from the knowledge space in Figure 1: $\{\} \subset \{a\} \subset \{a, b\} \subset \{a, b, d\} \subset \{a, b, c, d\}$. If any state of the knowledge structure is contained in at least one gradation, the knowledge space is said to be *well-graded*. The knowledge space in Figure 1 is well-graded. The four gradations can be represented by the corresponding order in which the items can be mastered: $abdc$, $abcd$, $acbd$, and $acdb$.

There are two concerns to be addressed regarding storage issues for this form of knowledge representation. For n items, there are 2^n possible knowledge states. However, in the example, there are only 8 out of 16 possible states in the knowledge space. There are $n!$ possible gradations, but in the example there are only 4 out of 24 possible gradations. In practice, there are much fewer states (and gradations) than the theoretical maximum. In the simplest case, if there is a simple order of the items (a Guttman-scale) yielding a single gradation (learning path) through the items, then there are only $n+2$ feasible states. In a study involving 50 items in high school mathematics, the size of the knowledge spaces obtained from systematically querying experts ranged from 900 to around 8,000 states—roughly the same order of magnitude across experts (Kambouri, Koppen, Villano, & Falmagne, 1991). The sizes of these knowledge space are far less than the theoretical maximum of $2^{50} \approx 10^{15}$.

The knowledge space forms the core of a knowledge assessment system. The goal of a knowledge assessment system is to locate, as efficiently and accurately as possible, a student's knowledge state in the knowledge structure. Stochastic knowledge assessment routines have been developed in which uncertainty regarding the student's knowledge state is represented by a probability distribution on the states (Falmagne & Doignon, 1988; Villano, 1991). To each state K in a knowledge structure K , we assign a probability $P(K)$. The assessment routine updates the probability distribution on the states to be consistent with the student's responses to a carefully chosen sequence of items. From the probability distribution on the states, we can also compute the probability of correct response to an item $\rho(q)$ as

$$\rho(q) = \sum_{K \in K_q} P(K)$$

where K_q is the set of states which contain item q . The probability of an incorrect response is $1 - \rho(q)$. Item parameters which can be estimated from stochastic learning models (Falmagne, 1989a; Villano, 1991; Falmagne, 1991a; Falmagne, 1991b) applied to empirical student data include the probability of a careless error and the probability of a lucky guess.

Student Model Construction

The two basic steps involved in the construction of a probabilistic student model are 1) building the structural relationships among the items and 2) determining the

Probabilistic Student Model

initial values for the probabilities in the models. Both of these steps rely on the judgments of experts or require empirical data from a population of students.

Building the Structural Model

Several methods for building the knowledge structure in KST have been explored:

1) Expert Judgments - an application of the QUERY Routine has been carried out by Kambouri, Koppen, Villano and Falmagne (1991). QUERY (Koppen and Doignon, 1990) is a computerized procedure designed to systematically question an experienced teacher/tutor and obtain the expert's "personal" knowledge structure. The results obtained for 50 items in high school mathematics revealed that the procedure could be applied in a realistic setting. Agreement among the experts was obtained on gross measures such as item difficulty, the relative size of the structures and the correlation of experts' responses given to the same questions posed by the routine. The limitations of this approach include a lack of agreement between the states which make up the experts' structures and the absence of an estimate of the distribution of the states for a population of students. The lack of agreement between the experts suggests that some of the experts differ concerning their ability to perform the task. Villano (1991) compared the performance of the individual experts' knowledge structures in computerized assessment routines and demonstrated a significant advantage in using some experts' structures over others.

2) Empirical Data - Villano (1991) investigated various methods for building knowledge structures. A refinement procedure was developed which involved the application of a probabilistic model to a large ($N=60,000$) reference set of students. Guttman-scale based structures were built by ordering items by increasing item difficulty to form a single learning path through the items. A third method utilized repeated applications of stochastic assessment routines to determine the collection of feasible states from the power set.

3) Neural Networks - A novel application of neural networks to construct a knowledge structure has been demonstrated by Harp, Samad and Villano (1992). Self-organizing feature maps are used to capture the possible states of student knowledge from an existing test database in the domain of aircraft fuel management. Noise-tolerance and insensitivity to feature map parameter values are demonstrated.

Initial Uncertainty of the Student Model

In the absence of empirical data (in the form of expert judgments or student responses) regarding the likelihood of knowledge states in a population of students, all the states would need to be considered equally likely. However, one of the significant benefits of probabilistic student models is the capability of incorporating knowledge about a population of students to improve the initial estimate of an individual student by the student model.

A variety of a priori probability distributions on the knowledge states have been studied by Villano (1991). The following distributions were implemented and evaluated in stochastic assessment routines:

1) *Uniform Prior* - all states in the structure are initially equiprobable.

Probabilistic Student Model

- 2) *Refined Prior* - the probabilities of the states are parameters estimated directly by applying a probabilistic model and maximum likelihood techniques to a large ($N=60,000$) reference set of student responses.
- 3) *Assessed Prior* - the probabilities of the states are estimated by taking the "average" of the final distributions resulting from the complete assessment of a large ($N=60,000$) reference set of student responses.

Falmagne (personal communication, November 1991) suggests that information regarding the background of the student, such as the student's age, level of education, prior training, etc. could also be used to "prime" the a priori probability distribution. This priming could be similar in spirit to the forward chaining that is often done at the start of some diagnostic expert systems.

Applications of the Student Model

Adaptive Assessment Item Selection

For an assessment routine to be adaptive, it must be capable of determining the next "best" question to pose to the student based on a dynamic student model. In KST, one method for selecting the most "informative" item to ask is choose the item with the least predictable response (Falmagne & Doignon, 1988; Villano, 1991). For the *half-split* item selection rule, we choose the item whose probability of being answered correctly, $\rho(q)$ is closest to .5. The reasoning is as follows. If $\rho(a) = .85$, then Item *a* would not be informative because it is almost certain the student would respond correctly. If $\rho(d) = .1$, then Item *d* is *not* informative because we are fairly certain ($1-.1 = .9$) that the student would fail this item. If $\rho(c) = .5$, then Item *c* would be the most informative item to ask of these three because there would be an equal chance of the student passing or failing item *c*. Item *c* is thus the item for which our estimate of how the student will respond is the most uncertain. (If two or more items are equally informative, then we randomly choose from among those items.) There is an entropy-based rule in KST in which we try to select the item which will bring about the greatest reduction in the entropy of the probability distribution on the states, but it has been shown to be equivalent to the half-split question selection rule under certain conditions (Falmagne and Doignon, 1988).

Adaptive Assessment Updating Routine

A dynamic student model would necessarily require periodic updating with each new item response obtained from the student. In order to perform adaptive assessment, an updating rule must be specified to maintain the current estimate of the student's performance.

In the stochastic knowledge assessment routines of Falmagne and Doignon (1988; Villano, 1991) the probability distribution on the states is maintained through an application of an updating rule which modifies the probabilities of the states to be consistent with each new response obtained from the student. For example, if the student responds correctly to an item, then the probability of the states which contain that item are increased, while the probability of the states which do not contain that item are decreased. Various updating rules are possible. The

Probabilistic Student Model

multiplicative updating rule specifies an operator (greater than 1) which is used to increase (by multiplying and then normalizing) the probability of the states consistent with the student's response. The larger the value of the parameter, the greater the change in the distribution. The calibration of such a parameter has been demonstrated by Villano (1991).

The multiplicative operator can be indexed by the item asked and the response given (correct or incorrect). Thus, a correct response to a particularly diagnostic item could have a stronger effect on the change in the mass of the probability distribution than some other less diagnostic item. (For example, items with low probability of a careless error could have higher update parameters. The response to these items would be judged to be more reliable due to the lower error rate.) During the instructional phase of an ITS, incorrect responses may be more prevalent as the student may be less cautious than during testing. Therefore, incorrect responses could have lower updating parameters set to exert less influence on the change in the distribution during the instructional phase. The multiplicative updating rule can be regarded as a generalization of a Bayesian updating rule as pointed out by Koppen in Falmagne and Doignon (1988).

Knowledge Type Representations

Both declarative and procedural knowledge can be represented and integrated in a knowledge structure. A distinction between these two traditional knowledge types may be necessary for expository purposes (lesson presentation may differ for teaching declarative vs. procedural knowledge) as well as for testing formats. Procedural knowledge may be tested by presenting a task for the student to complete and monitoring the student's performance. Declarative knowledge, although implicitly tested during the completion of a task, can be assessed directly using standard fill-in or multiple choice questions. In order to satisfy these and other concerns for maintaining the distinction between declarative and procedural knowledge, the complete knowledge structure encompassing both can be divided in to two "substructures" as indicated in Figure 2.

Curriculum Representation

Various learning paths through a curriculum may be represented in a student model to accommodate different instructional strategies of educators and different learning styles on the part of students. If an item has more than one unique set of prerequisites, then alternative paths through the items should be represented. The curriculum path determines the next lesson (associated with an item) to teach in a directed, as opposed to an exploratory ITS.

In KST, the learning paths (called gradations) may be used to guide the teaching of the student. In a well-graded knowledge space, the next lesson to teach is the one tested by the next item in the learning path. In the event that there is more than one path to follow from the current knowledge state, you may choose the "easiest" item (the item with the highest probability of being answered correctly), or the item along the most traveled (i.e., most probable) learning path. Additional parameters which should affect teaching include the history of the knowledge state over time and an estimate of the learning rate of the student. The learning rate of the student can be estimated using the stochastic learning path model specified by Falmagne, (1991b).

Probabilistic Student Model

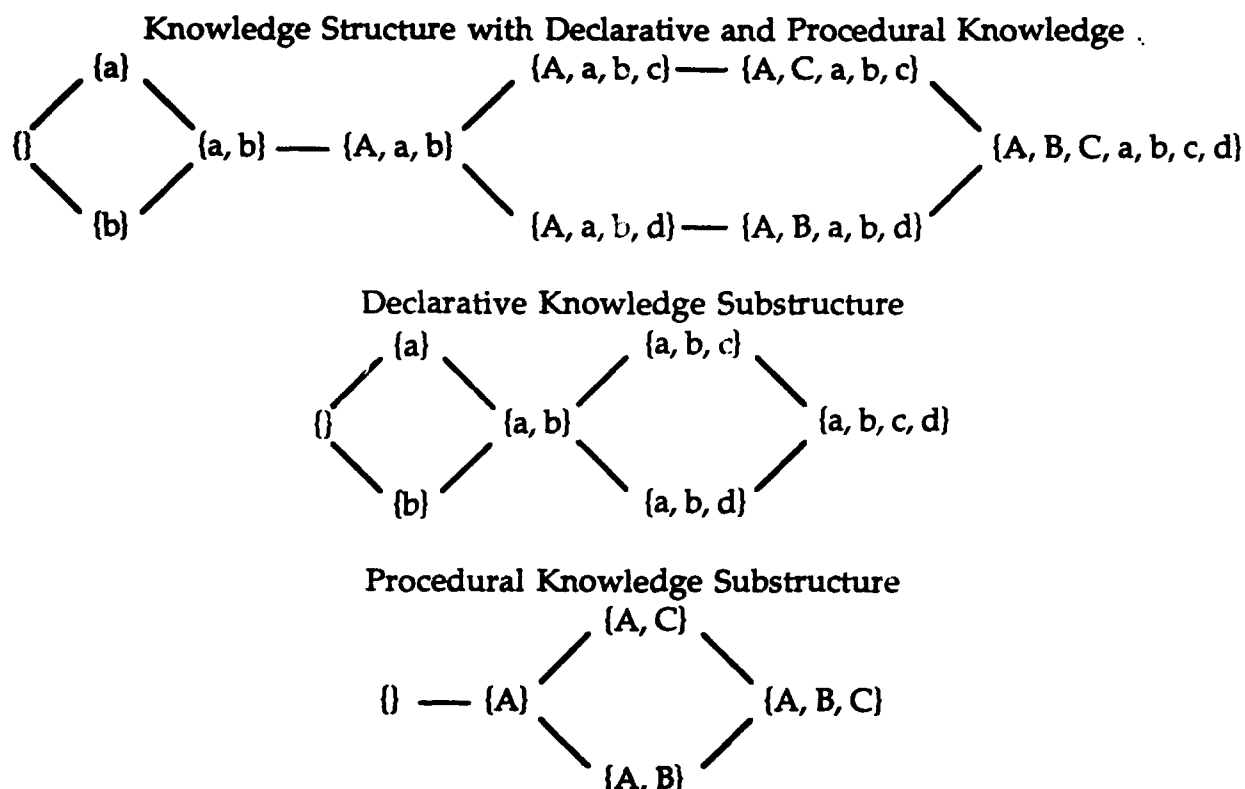


Figure 2. An example of a knowledge structure and two of its substructures. The declarative items appear as small letters. The capital letters denote procedural items.

If all the parameters of the model have been estimated for a population of students, then we can re-estimate the learning rate parameter for a particular student. If we observe n patterns of responses at different times, t_1, t_2, \dots, t_n , then we can estimate the students' learning rate by maximizing the likelihood of the student's learning rate parameter at time t_n .

Hint Level Selection

The content of a hint or explanation in an ITS relies upon the student model's representation of the student's level of mastery of the material. Advice or help to be presented to the student should be tailored to the individual student's needs. Advanced students may prefer to be given more terse explanations, whereas novice students could be given more elaborate guidance. The coaching which a student receives from the ITS should be careful only to include references to concepts which the student model indicates as having been mastered.

The level of explanation or hints in KST could depend upon whether the student is being assessed at a coarse substructure or at a finer, more diagnostic level. The height of an item in the knowledge structure is a rough measure of item difficulty and could also be used to determine the level of hinting. The height of an item $h(q)$ is defined as the smallest number of items which must be mastered before q . (Kambouri et al., 1991). In the example knowledge space in Figure 1, $h(a) = 0$ and $h(d) = 2$. (There are no items which must be mastered before Item a and at least 2 items (a, b or a, c) which must be mastered before Item d . If the items span a wide

Probabilistic Student Model

range of heights, then each level of help could be associated with a particular height interval. For example, if we wish to offer 3 levels of help in an ITS (beginner, novice, advanced) and the heights for 20 items range from 0 to 12, then Table 1 shows one possible mapping of the item height to the hint level.

Table 1

Definition of Help Level from Item Height

Item Height	Help Level
0 to 3	beginner
4 to 8	novice
9 to 12	advanced

An item parameter such as the probability of a careless error may also influence hinting. For example, if an item had a relatively high probability of a careless error, a hint might warn the student to take extra time to check and confirm the answer to the item.

Advancement Criterion

A student's advancement through the curriculum may need to be directed or at least monitored by the ITS, particularly in domains such as mathematics, where advanced concepts will not be easily mastered without a strong understanding of fundamental principles or prerequisites. (A debate comparing directed vs. exploratory learning is well beyond the scope of this paper.)

The student would be expected to master the current item in the learning path before moving on to more advanced items. If this constraint is relaxed, a criterion could be specified to control advancement. Thus, mastery of an item could be defined by a score on equivalence class of items. In addition, a minimum number of instances of an item may be required to which the student must respond.

Student Feedback

An inspectable, detailed representation of the learner's mastery of the material can provide feedback regarding the student's most recent accomplishments and most pressing weaknesses.

In KST, rather than reporting a single score (i.e., ability = 95%), we can be much more specific and indicate the most advanced item that has been mastered as well as a list of the missing items and/or future items to be mastered. If a single score is preferred, we should not just use "blind" averaging of the scores on the items, but rather take advantage of diagnostic information for the specific items. For example, weighting the average score by the heights of the items. Ideally, we would prefer not to lose the distinction between a student who can answer many simple items versus one who can answer a few difficult items. A proposed project that has been on the table for a number of years in Falmagne's lab (personal communication) involves the generation of diagnostic dialogue once a student's knowledge state has been isolated. The following issues would need to be addressed in the development of such a system:

Probabilistic Student Model

- 1) an analysis of the items in terms of skills or concepts should be performed.
- 2) quantitative aspects of skill should be translated into linguistic terms.
- 3) degrees of doubt regarding the diagnosis should be expressed.
 "Student probably knows multiplication, but may be weak in percents."
- 4) broad issues in generating discourse would need to be addressed.

Evaluation of a Probabilistic Student Model

An important consideration for utilizing probabilistic student models in an ITS is the ability to quantitatively evaluate their effectiveness using established statistical techniques on simulated and real student data. Some points to consider when evaluating student models are given below:

Error sensitivity. The responsiveness of the student model to careless errors or lucky guesses on the part of the student is a critical feature of probabilistic student models and should be carefully studied.

Parameter sensitivity. How critical to the success of the student model are the initial parameter estimates? The importance of the estimates of the a priori probability distribution on the states and the item parameters has to be reviewed.

Efficiency. Ideally, a student model should minimize the number of questions that are necessary to obtain an accurate assessment of the student. The cost of asking additional items should be measured against any increase in assessment accuracy.

Learning rate. How quickly the student model converges to an accurate estimate of the state of the student's knowledge is also of interest and related to the issue of efficiency.

Assessment accuracy. Many of the above considerations rely upon some measure of the quality of the assessment. One such measure involves computing a prediction index (Villano, 1991) which represents the proportion of student responses correctly predicted for items which have not yet been asked during the assessment.

Discussion

Knowledge Space Theory was developed to conduct efficient, computerized student assessments and therefore may be a viable choice as a probabilistic student model. A variety of techniques have been investigated for building the structural and probabilistic components of the student model in KST. A realistic concern regarding the implementation of knowledge spaces is the possible combinatorial explosion in terms of the size of a knowledge space if there is a significant lack of structure among the items. The size of a knowledge structure may not be an important consideration with the rapid increases in the power and storage capacity of modern computers. A large number of research issues remain to be explored in regards to applying probabilistic student models to an Intelligent Tutoring System. However, the goal of developing a dynamic, non-deterministic student model capable of robust, individualized assessment may be well worth the cost.

References

- Degreef, E., Doignon, J.-P., Ducamp, A., & Falmagne, J.-C. (1986). Languages for the assessment of knowledge. *Journal of Mathematical Psychology*, 30, 243-256.
- Doignon, J.-P., & Falmagne, J.-C. (1985). Spaces for the assessment of knowledge. *International Journal of Man-Machine Studies*, 23, 175-196.
- Doignon, J.-P., & Falmagne, J.-C. (1988). Parametrization of knowledge structures. *Discrete Applied Mathematics*, 21, 87-100.
- Falmagne, J.-C. (1991a). *Stochastic Learning Paths in a Knowledge Structure*. (Tech. Rep. No. MBS 91-17). Irvine: University of California, Irvine Research Unit in Mathematical Behavioral Sciences.
- Falmagne, J.-C. (1991b). *Notes on Six Markov Learning Models for Knowledge Structures*. Unpublished manuscript.
- Falmagne, J.-C. (1989a). A latent trait theory via a stochastic learning theory for a knowledge space. *Psychometrika*, 54, 283-303.
- Falmagne, J.-C. (1989b). Probabilistic knowledge spaces: A review. In F. Roberts (Ed.), *Applications of combinatorics and graph theory to the biological and social sciences* (IMA Vol. 17). New York: Springer-Verlag.
- Falmagne, J.-C. & Doignon, J.-P. (1988). A class of stochastic procedures for the assessment of knowledge. *British Journal of Mathematical and Statistical Psychology*, 41, 1-23.
- Falmagne, J.-C., Koppen, M., Villano, M., Doignon, J.-P., Johannesen, L. (1990). Introduction to knowledge spaces: How to build, test, and search them. *Psychological Review*, 97(2), 201-224.
- Harp, S. A., Samad, T., & Villano, M. (1992) *Modeling student knowledge with self-organizing feature maps*. Manuscript submitted for publication.
- Kambouri, M., Koppen, M., Villano, M., & Falmagne, J.-C. (1991). *Knowledge assessment: Tapping human expertise by the QUERY routine*. Manuscript submitted for publication.
- Koppen, M., & Doignon, J.-P. (1990). How to build a knowledge space by querying an expert. *Journal of Mathematical Psychology*, 34, 311-331.
- Villano, M., Falmagne, J.-C., Johannesen, L. & Doignon, J.-P. (1987). Stochastic procedures for assessing an individual's state of knowledge.

Probabilistic Student Model

Proceedings of the International Conference on Computer Assisted Learning in Post-Secondary Education (pp. 369-371). Calgary, Alberta, Canada: University of Calgary Press.

Villano, M. (1991). *Computerized Knowledge Assessment: Building the knowledge structure and calibrating the assessment routine*. (Doctoral dissertation, New York University, New York, 1991). *Dissertation Abstracts International*, 52-12B.